

Constraints on the Electrical Charge Asymmetry of the Universe

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Abstract. We use the isotropy of the Cosmic Microwave Background to place stringent constraints on a possible electrical charge asymmetry of the universe. We find the excess charge per baryon to be $q_{e-p} < 10^{-26}e$ in the case of a uniform distribution of charge, where e is the charge of the electron. If the charge asymmetry is inhomogeneous, the constraints will depend on the spectral index, n , of the induced magnetic field and range from $q_{e-p} < 5 \times 10^{-20}e$ ($n = -2$) to $q_{e-p} < 2 \times 10^{-26}e$ ($n \geq 2$). If one could further assume that the charge asymmetries of individual particle species are not anti-correlated so as to cancel, this would imply, for photons, $q_\gamma < 10^{-35}e$; for neutrinos, $q_\nu < 4 \times 10^{-35}e$; and for heavy (light) dark matter particles $q_{\text{dm}} < 4 \times 10^{-24}e$ ($q_{\text{dm}} < 4 \times 10^{-30}e$).

1. Introduction

With the substantial improvement of experimental and observational techniques in particle physics and astrophysics, it has become possible to test some of the assumptions that go into constructing models of fundamental interactions and the universe. The aim of this work is to place a cosmological constraint on the presence of an electrical charge asymmetry in the universe. The first cosmological analysis of a charged universe undertaken was in the context of the Steady State model of the universe by Bondi & Lyttleton in 1959 [1], where an attempt was made to explain the recession of the galaxies through electromagnetic repulsion. To implement charge non-conservation, Maxwell's equations were modified to include a direct coupling to the vector potential, violating gauge invariance. Swann [2] confuted this analysis; Barnes [3] correctly reanalysed a similar model, using the Proca theory of electromagnetism. The spontaneous breaking of gauge symmetry, and the subsequent development of a charge imbalance, was considered by Ignatiev *et al.* [4], and Dolgov & Silk [5] used it as a mechanism of creation of primordial magnetic fields. An implementation of a homogeneous and isotropic charge density was proposed in the context of massive electrodynamics [6], and the possibility of charge non-conservation has been analysed in brane world models [7], and in varying speed of light theories [8].

In principle, a potential charge asymmetry could be carried by a number of different, stable components in the universe. These can include: photons, neutrinos, dark matter, or a difference in the electron and proton charges. Experimental constraints, based on the lack of dispersion from pulsar signals, limit the photon charge to be $q_\gamma < 5 \times 10^{-30}e$ [9] (a less restrictive laboratory constraint, $q_\gamma < 8 \times 10^{-17}e$, has also been established [10]). The analysis of the luminosity evolution of red giants in globular clusters leads to a constraint on the charge of neutrinos (or the charge of any sub-keV particle) of $q_\nu < 2 \times 10^{-14}e$ [11]. Direct, laboratory constraints based on gas-efflux measurements [12], electro-acoustic techniques [13], and Millikan-type experiments using steel balls [14], all limit the electron-proton charge difference to be $q_{e-p} < 10^{-21}e$. For particles generally with masses less than ~ 1 MeV, Big Bang Nucleosynthesis rules out charges greater than $10^{-10}e$ [15]. Concerning dark matter, constraints have been established on the fraction of dark matter due to charged heavy particles of the order of 10^{-5} [16].

In this work we will obtain general constraints on a possible charge asymmetry, characterised by a uniform or a stochastic distribution, placing the potential contribution of individual components in context. The premise will be that charge was generated at some very early time, but has been conserved in the period in which we are establishing constraints.

We use Heaviside-Lorentz electromagnetic units with $e = \sqrt{4\pi\alpha}$, $c = 1 = 8\pi G$; Greek indices run from 0 to 3 and Latin indices from 1 to 3. The scale factor is normalised to $a(\tau_0) = 1$ today, where τ denotes conformal time.

2. Model of a charged universe

In this analysis we use the 1+3 covariant formalism developed in [17]: we consider a general class of homogeneous space-times for which it is possible to define a preferred velocity field, u^α , determining the fundamental fluid flow lines and satisfying $u^\alpha u_\alpha = -1$. The existence of this vector field generates a unique splitting of spacetime, given by the instantaneous three-dimensional rest space of an observer moving with 4-velocity u^α and the one-dimensional space u^α itself. The metric tensor of this spacetime, $g_{\alpha\beta}$ can be written like $g_{\alpha\beta} = h_{\alpha\beta} - u_\alpha u_\beta$, where $h_{\alpha\beta}$ is a projection tensor and represents effectively the spatial metric for the observer moving with u^α . Any physical tensor field on this spacetime can be separated with respect to $h_{\alpha\beta}$ and u^α into space and time parts. The covariant derivative of a tensor field $S^{\alpha\beta}$ splits into a comoving time derivative $\dot{S}^{\alpha\beta} = u^\gamma S^{\alpha\beta}_{;\gamma}$ and a covariant spatial derivative $D_\gamma S^{\alpha\beta} = h_\gamma^\delta h^\alpha_\mu h^\beta_\nu S^{\mu\nu}_{;\delta}$. In particular, the splitting of the covariant derivative of the 4-velocity takes the form:

$$u_{\alpha;\beta} = \sigma_{\alpha\beta} + \omega_{\alpha\beta} + \frac{1}{3}\Theta h_{\alpha\beta} - \dot{u}_\alpha u_\beta, \quad (1)$$

where $\sigma_{\alpha\beta}$ is the shear tensor, $\omega_{\alpha\beta}$ is the vorticity tensor, $\Theta = 3\dot{a}/a$ is the volume expansion, and $\dot{u}_\alpha = u_{\alpha;\beta} u^\beta$ is the acceleration vector. The electromagnetic field tensor $F_{\alpha\beta} = -F_{\beta\alpha}$ is split into electric and magnetic fields as measured by an observer moving with u^α :

$$F_{\alpha\beta} = 2u_{[\alpha} E_{\beta]} + \epsilon_{\alpha\beta\gamma} B^\gamma, \quad (2)$$

where the electric and magnetic 3-vectors are given by $E_\alpha = F_{\alpha\beta} u^\beta$, $B_\alpha = \frac{1}{2} \epsilon_{\alpha\beta\gamma} F^{\beta\gamma}$, and $\epsilon_{\alpha\beta\gamma} = \eta_{\alpha\beta\gamma\delta} u^\delta$ is the volume element for the orthogonal rest space $h_{\alpha\beta}$. We define the current density 4-vector j^α , where $q = -j^\alpha u_\alpha$ is the charge density, and $J^\alpha = h^\alpha_\beta j^\beta$ is the spatial current measured by u^α . Then, Maxwell's equations $F^{\alpha\beta}_{;\beta} = j^\alpha$, $F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = 0$ become [17, 18]

$$D_\alpha E^\alpha + 2\omega_\alpha B^\alpha = q \quad (3)$$

$$D_\alpha B^\alpha - 2\omega_\alpha E^\alpha = 0 \quad (4)$$

$$h_{\alpha\beta} \dot{E}^\beta - \epsilon_{\alpha\beta\gamma} D^\beta B^\gamma = -J_\alpha + \epsilon_{\alpha\beta\gamma} \dot{u}^\beta B^\gamma + \left(\omega_{\alpha\beta} + \sigma_{\alpha\beta} - \frac{2}{3}\Theta h_{\alpha\beta} \right) E^\beta \quad (5)$$

$$h_{\alpha\beta} \dot{B}^\beta + \epsilon_{\alpha\beta\gamma} D^\beta E^\gamma = -\epsilon_{\alpha\beta\gamma} \dot{u}^\beta E^\gamma + \left(\omega_{\alpha\beta} + \sigma_{\alpha\beta} - \frac{2}{3}\Theta h_{\alpha\beta} \right) B^\beta, \quad (6)$$

where $\omega_\alpha = \frac{1}{2} \epsilon_{\alpha\beta\gamma} \omega^{\beta\gamma}$ is the vorticity vector. It appears that the motion of the observer affects the form of Maxwell's equations: beside the usual spatial divergences, curls and time derivatives of the fields, one has the appearance of extra terms representing the motion of the family of fundamental observers moving with 4-velocity u^α . Moreover, the current conservation equation $j^\alpha_{;\alpha} = 0$ takes the form

$$\dot{q} + \Theta q + D_\alpha J^\alpha + \dot{u}_\alpha J^\alpha = 0, \quad (7)$$

and the covariant form of Ohm's law is

$$j_\alpha + u_\alpha u_\beta j^\beta = \sigma F_{\alpha\beta} u^\beta, \quad (8)$$

where σ denotes the conductivity. Projecting onto the instantaneous rest space of the fundamental observer, Ohm's law becomes $J_\alpha = \sigma E_\alpha$.

We consider a model of the universe filled with a charged perfect fluid, which could be matter or radiation, characterised by the energy-momentum tensor $T_F^{\alpha\beta} = \rho u^\alpha u^\beta + p h^{\alpha\beta}$; the charge will give rise to an electromagnetic field, with $T_{EM}^{\alpha\beta} = F^\alpha{}_\gamma F^{\beta\gamma} - \frac{1}{4} g^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}$. The energy-momentum conservation equation accounts for the interaction between the fluid and the field, so that [19]

$$T_F^{\alpha\beta}{}_{;\beta} = F^{\alpha\beta} j_\beta. \quad (9)$$

Using the above definitions, the coupling term can be rewritten as

$$F^{\alpha\beta} j_\beta = q E^\alpha + F^{\alpha\beta} J_\beta = q E^\alpha + u^\alpha E^\beta J_\beta + \epsilon^{\alpha\beta\gamma} J_\beta B_\gamma. \quad (10)$$

We further assume the usual equation of state for a perfect fluid $p = w\rho$; then, the energy conservation equation for the fluid, $u_\alpha T_F^{\alpha\beta}{}_{;\beta} = u_\alpha F^{\alpha\beta} j_\beta$, takes the form

$$\dot{\rho} + (1+w)\Theta\rho = -E^\beta J_\beta, \quad (11)$$

and the momentum conservation equation $h_{\alpha\beta} T_F^{\beta\gamma}{}_{;\gamma} = h_{\alpha\beta} F^{\beta\gamma} j_\gamma$ takes the form

$$(1+w)\rho \dot{u}_\alpha + w D_\alpha \rho = q E_\alpha + \epsilon_{\alpha\beta\gamma} J^\beta B^\gamma. \quad (12)$$

If the charged fluid is composed by non-relativistic matter, one defines $q = \varepsilon e n_{\text{mat}} = \frac{\varepsilon e}{m} \rho_{\text{mat}}$, where εe and n_{mat} represent respectively the charge and the number density of the fluid particles, and m is their mass; if instead the radiation component is charged, one has $q = \varepsilon e n_{\text{rad}} \propto \rho_{\text{rad}}^{3/4}$, with similar definitions. It appears from this picture that the introduction of a charged fluid breaks the isotropy of the universe, through the creation of currents and electromagnetic fields. Therefore, it is in principle possible to use the CMB as a measure of the isotropy of the universe to constrain the presence of an overall charge asymmetry.

We must further consider the fact that the universe has a very high conductivity, during all the phases of its evolution. The conductivity varies in the different epochs of the evolution of the universe, depending on the number of charge carriers and on the dominant scattering process. Before e^+e^- annihilation, the conductivity has been evaluated in [20], and is found to decay with temperature like $\sigma \sim T/(\alpha^2 \log(1/\alpha))$, where α is the fine structure constant. During the radiation dominated era, considering Thomson scattering and proceeding as in [21], one finds $\sigma = 2\pi \frac{n_e}{n_\gamma} \frac{m_e}{e^2} \simeq 3 \times 10^{13} \text{ sec}^{-1}$. During the matter dominated epoch, the dominant scattering process is again Thomson scattering of CMB photons with the residual free electrons, and one has $\sigma \sim 3 \times 10^{10} \text{ sec}^{-1}$ [22]. We perform our analysis in the ideal magnetohydrodynamics limit, for which the conductivity goes to infinity while the current remains finite [19]. In the reference frame of comoving observers, corresponding to the reference frame of the fluid in this picture, Ohm's law takes the form $J_\alpha = \sigma E_\alpha$. Consequently, applying the magnetohydrodynamic limit, one finds that the electric field (but not the magnetic field) must go to zero [23]. This ensures that the spatial current remains finite, avoiding the possibility of an instantaneous response of the fluid to the electromagnetic fields. Note

that this is valid because our analysis is performed in the reference frame of observers comoving with the fluid, characterised by the 4-velocity u^α ; in a reference frame with 4-velocity $\tilde{u}^\alpha = u^\alpha - v^\alpha$, the magnetohydrodynamic limit implies the presence of an electric field $\tilde{E}_\alpha = -\epsilon_{\alpha\beta\gamma}v^\beta\tilde{B}^\gamma$.

Another way of rephrasing the magnetohydrodynamic limit is found by considering the time evolution of the electric and magnetic fields given by Maxwell's equations (5) and (6). The presence of a primordial, uniform magnetic field in a Friedmann-Robertson-Walker (FRW) universe is accounted for by assuming that the energy density of the field is a first order quantity in perturbation theory, and that the field itself is a small background component. We introduce the presence also of an electric field with the same characteristics. Then, the evolution of these fields at the lowest order is given by

$$\dot{B}_\alpha = -\frac{2}{3}\Theta B_\alpha, \quad (13)$$

$$\dot{E}_\alpha = -J_\alpha - \frac{2}{3}\Theta E_\alpha. \quad (14)$$

The first of these equations suggests that the magnetic field varies on an approximative time-scale given by the Hubble time Θ^{-1} . By substituting with Ohm's law $J_\alpha = \sigma E_\alpha$ in the second equation, one finds that the time-scale of variation of the electric field is instead of the order $(\sigma + (2/3)\Theta)^{-1}$. One further has that, for all the epochs of the evolution of the universe, $\sigma/\Theta \gg 1$ (for example, $\sigma/\Theta \sim 10^{28}$ today). Therefore, while the magnetic field varies on a time-scale comparable to the Hubble time, the electric field varies on a much smaller time-scale, given by σ^{-1} , and gets dissipated much quicker. The infinite conductivity limit provides an explanation of the reason why large scale magnetic fields, and not electric fields, are observed in the universe.

In the case of a charged cosmic fluid which we are analysing, and in the reference frame of comoving observers, adopting the infinite conductivity limit reduces the form of the relevant equations to

$$\dot{\rho} + (1+w)\Theta\rho = 0, \quad (15)$$

$$(1+w)\rho\dot{u}_\alpha + wD_\alpha\rho = \epsilon_{\alpha\beta\gamma}J^\beta B^\gamma, \quad (16)$$

for the energy and momentum conservation, and

$$2\omega_\alpha B^\alpha = q \quad (17)$$

$$D_\alpha B^\alpha = 0 \quad (18)$$

$$\epsilon_{\alpha\beta\gamma}D^\beta B^\gamma = J_\alpha - \epsilon_{\alpha\beta\gamma}\dot{u}^\beta B^\gamma \quad (19)$$

$$h_{\alpha\beta}\dot{B}^\beta = \left(\omega_{\alpha\beta} + \sigma_{\alpha\beta} - \frac{2}{3}\Theta h_{\alpha\beta}\right)B^\beta, \quad (20)$$

for Maxwell's equations. We have defined the charge density as $q = \varepsilon e n$, both in the case of charged non-relativistic matter and radiation; therefore, using $\rho_{\text{mat}} \propto n_{\text{mat}}$ and $\rho_{\text{rad}} \propto n_{\text{rad}}^{4/3}$, Eq. (15) implies the charge evolution $\dot{q} + \Theta q = 0$. Equation (17) shows that in a universe with infinite conductivity, the presence of a charge density implies the presence of a magnetic field, and induces vorticity in the metric. Therefore, the

analysis in a non-perturbative framework should be carried on in the context of tilted Bianchi universes, *i.e.* homogeneous and anisotropic universes in which the surfaces of homogeneity are not orthogonal to the matter flow. However, the goal of this analysis is to constrain the presence of charge in the universe by using the isotropy of the CMB. Consequently, the charge has to be considered a small perturbation, $\varepsilon \rightarrow 0$, and we want to use linear perturbation theory on a FRW universe. The relevant equation at this purpose is (17): a constraint on the charge density can be derived from constraints on the magnetic field and the vorticity induced by it. We need to evaluate the time evolution of these two quantities. At the lowest order, the magnetic field evolution is

$$\dot{B}_\alpha + \frac{2}{3}\Theta B_\alpha = 0, \quad (21)$$

and the vorticity evolution is [24, 25]

$$\dot{\omega}_\alpha + \frac{2}{3}\Theta \omega_\alpha = -\frac{1}{2}\epsilon_{\alpha\beta\gamma} D^\beta \dot{u}^\gamma. \quad (22)$$

Substituting with the momentum conservation equation (16), and using the identity $\epsilon_{\alpha\beta\gamma} D^\beta D^\gamma f = -2\dot{f}\omega_\alpha$ [24], equation (22) becomes

$$\dot{\omega}_\alpha + \left(\frac{2}{3} - w\right) \Theta \omega_\alpha = -\frac{1}{2\rho(1+w)} \epsilon_{\alpha\beta\gamma} D^\beta \epsilon^\gamma_{\delta\mu} J^\delta B^\mu. \quad (23)$$

The magnetic field evolution equation gives the usual scaling behaviour $B_\alpha = B_{0\alpha}/a^2$, where B_0 denotes the field amplitude today. The evolution for the vorticity is more involved. We know however that the charge density evolves like the number density of particles, $\dot{q} + \Theta q = 0$: this equation can give us insight on the behaviour of vorticity. By deriving Eq. (17), and imposing the charge scaling, $2\dot{\omega}_\alpha B^\alpha + 2\omega_\alpha \dot{B}^\alpha = -\Theta q$, we find, from the magnetic field evolution

$$B^\alpha \dot{\omega}_\alpha = -\frac{1}{3}\Theta B^\alpha \omega_\alpha. \quad (24)$$

This identity is satisfied in the two cases $(a\omega_\alpha)^\cdot = 0$, and $(a\boldsymbol{\omega})^\cdot \perp \mathbf{B}$. In the first case, the evolution of the vorticity is such that $\dot{\omega}_\alpha + \frac{\dot{a}}{a}\omega_\alpha = 0$; in the second case, one has that only the component parallel to the magnetic field satisfies $\dot{\omega}_\parallel + \frac{\dot{a}}{a}\omega_\parallel = 0$. In the following, we will always assume that $\omega_\alpha \propto a^{-1}$. Given that we want to constrain the charge using Eq. (17), we are in fact interested only in ω_\parallel . However, in the following we will set constraints on the vorticity vector independently, through the induced CMB anisotropies: our assumption, therefore, corresponds to postulating that the entire vorticity contribution to CMB anisotropies comes only from the ω_\parallel component. A premise that is conservative, in the sense that it makes the final limit on the charge less tight.

We now proceed to evaluate the bounds on the charge in two different configurations: uniformly and stochastically distributed charge. The strategy of limiting the charge asymmetry of the universe by using the observed isotropy of the FRW spacetime has already been adopted in two previous works: Orito and Yoshimura [26] also used the measurement of CMB temperature fluctuations, instead Masso and Rota

[27] derived a bound using Nucleosynthesis. In both these references, however, it is claimed that the presence of a non-zero charge density would generate a large scale electric field in the universe: they modelled the universe as an insulating medium rather than a highly conducting one. Our work differs from both of them in this, we believe, crucial aspect.

3. Uniform distribution of charge

We first focus on a uniform charge distribution in a homogeneous spacetime. The anisotropic expansion of the universe will leave an imprint on the propagation of light rays from the last-scattering surface until today. In [28] tight constraints were derived on both ω and B for a general class of homogeneous, anisotropic models. It was found that

$$B(\tau_0) < 3 \times 10^{-9} \Omega^{1/2} h \text{ Gauss}, \quad (25)$$

$$\omega(\tau_0) < 10^{-7} H_0, \quad (26)$$

where $H_0 = 100 h \text{ km sec}^{-1} \text{ Mpc}^{-1}$ is the Hubble constant today, and $1 - \Omega$ is the curvature. The bound in Eq. (26) was found assuming the usual scaling for the vorticity in the matter era in the absence of sources: $\omega_\alpha = \omega_{0\alpha}/a^2$, as can be derived from Eq. (23). Therefore, we have to correct this bound accounting for our assumption for the evolution of vorticity sourced by the charge: $\omega_\alpha = \omega_{0\alpha}/a$. This simply changes the limit by a factor of $1/a(\eta_{\text{rec}}) \simeq 10^3$: $\omega_0 < 10^{-4} H_0$. With these constraints we obtain a limit on the charge asymmetry using the Schwarz inequality on Eq. (17),

$$q \leq 2|\omega(\tau_0)||B(\tau_0)| \leq 2.4 \times 10^{-74} h^2 \Omega^{1/2} \text{ GeV}^3. \quad (27)$$

It may be that the charge asymmetries corresponding to individual particle species are such as to cancel to some extent, yielding to a universe which is more neutral overall. However, should this not be the case, this last equation imply a maximal allowed charge $q_X = q/n_X$ for each different constituent X , where n_X is the number density of charged particles. For a flat universe ($\Omega = 1$), and assuming that the charge is due to a difference between the charges of the electron and the proton, we divide by the baryon density of the universe $\Omega_B h^2 = 0.02$ to find

$$q_{e-p} \lesssim 10^{-26} e. \quad (28)$$

If the charged particles are instead dominated by the dark matter constituents, we get $q_{\text{dm}} \lesssim 4 \times 10^{-27} e m_{\text{dm}} \text{ GeV}^{-1}$. For example, a conservative limit comes from neutralinos of mass of 1 TeV, $q_{\text{dm}} \lesssim 4 \times 10^{-24} e$; considering instead a light dark matter particle with $m_{\text{dm}} \sim 10 \text{ MeV}$ [29], one gets $q_{\text{dm}} \lesssim 4 \times 10^{-30} e$. If we assume that the charge imbalance is predominantly caused by photons, with $n_\gamma = 421.84 \text{ cm}^{-3}$, we obtain $q_\gamma \lesssim 10^{-35} e$; for each species of neutrinos we obtain instead $q_\nu \lesssim 0.4 \times 10^{-34} e$, from $n_\nu = 115.05 \text{ cm}^{-3}$. All these constraints are orders of magnitude more restrictive than the limits from laboratory experiments or astrophysical observations mentioned above, subject to the assumption that any charge asymmetries between species are not anti-correlated.

4. Stochastic distribution of charge

One might expect that any mechanism of charge creation will be local, possibly a result of local violation of gauge invariance arising at some phase transition, or due to the escape of charge into extra dimensions. It would then be more appropriate to consider a stochastic distribution of charge asymmetry with, or without, a net imbalance in charge. The simplest assumption we can make, is that the magnetic field and the vorticity created by the charge density are two independent stochastic variables, with Gaussian distribution. Homogeneity and isotropy forces the first moment of the distributions to be zero and hence $\langle q \rangle = 0$. We further assume that the magnetic field and vorticity induced have power law power spectra, $B^2(k) = B k^n$ and $\omega^2(k) = \Omega k^m$, where the two spectral indices are different consequently to the independence assumption. We have then that:

$$\begin{aligned}\langle B_i(\mathbf{k}) B_j^*(\mathbf{q}) \rangle &= \frac{(2\pi)^3}{2} \delta^3(\mathbf{k} - \mathbf{q}) (\delta_{ij} - \hat{k}_i \hat{k}_j) B^2(k), \quad k < k_c \\ \langle \omega_i(\mathbf{k}) \omega_j^*(\mathbf{q}) \rangle &= \frac{(2\pi)^3}{2} \delta^3(\mathbf{k} - \mathbf{q}) (\delta_{ij} - \hat{k}_i \hat{k}_j) \omega^2(k),\end{aligned}$$

where we need to introduce an upper cutoff frequency k_c in the magnetic field power spectrum, which accounts for the interaction of the magnetic field with the cosmic plasma at small scales; $k_c(\tau_{\text{rec}}) \simeq 10 \text{ Mpc}^{-1}$, from [30]. For the following calculations, we do not need to assume a cutoff frequency for the vorticity. The factors $(\delta_{ij} - \hat{k}_i \hat{k}_j)$ come from the divergence-free property of both the magnetic field and the vorticity [25]. We define $B_\lambda^2 = \langle B^i(\mathbf{x}) B_i(\mathbf{x}) \rangle|_\lambda$ and $\omega_\lambda^2 = \langle \omega^i(\mathbf{x}) \omega_i(\mathbf{x}) \rangle|_\lambda$ to be the energy densities of the magnetic field and the vorticity in a region of size λ .

We wish to estimate the mean fluctuation of charge in a region of size λ :

$$\langle q_\lambda^2 \rangle = \frac{1}{V_\lambda^2} \int d^3 r_1 d^3 r_2 e^{-\frac{r_1^2}{2\lambda^2}} e^{-\frac{r_2^2}{2\lambda^2}} \langle q(\mathbf{r}_1 + \mathbf{x}) q(\mathbf{r}_2 + \mathbf{x}) \rangle.$$

From Eq. (17) one has

$$q(\mathbf{k}) = \frac{2}{(2\pi)^3} \int d^3 p \omega_i(\mathbf{k} - \mathbf{p}) B^i(\mathbf{p}), \quad (29)$$

so the charge density power spectrum is

$$\langle q(\mathbf{k}) q^*(\mathbf{h}) \rangle = \Omega B \delta^3(\mathbf{k} - \mathbf{h}) \int d^3 p |\mathbf{k} - \mathbf{p}|^m p^n [1 + \widehat{(\mathbf{k} - \mathbf{p}) \cdot \hat{\mathbf{p}}}^2], \quad (30)$$

with $0 < p < k_c$. We obtain an analytical approximation:

$$\langle q_\lambda^2 \rangle \simeq \frac{4 B_\lambda^2 \omega_\lambda^2}{\Gamma(\frac{n+3}{2}) \Gamma(\frac{m+3}{2})} f(n, m, \lambda, k_c), \quad (31)$$

with

$$\begin{aligned}f &= \frac{2(\lambda k_c)^{n+3}}{3(n+3)} \Gamma\left(\frac{m+3}{2}, (\lambda k_c)^2\right) + \frac{(\lambda k_c)^{n+m+3}}{n+m+3} \times \\ &\quad \left[\Gamma\left(\frac{3}{2}\right) - \Gamma\left(\frac{3}{2}, (\lambda k_c)^2\right) \right] + \frac{2m-n-3}{3(n+3)(n+m+3)} \times \\ &\quad \left[\Gamma\left(\frac{n+m+6}{2}\right) - \Gamma\left(\frac{n+m+6}{2}, (\lambda k_c)^2\right) \right],\end{aligned}$$

where $\Gamma(a, x)$ denotes the incomplete gamma function (6.5.3 of [31]).

Eq. (31) provides a limit for the charge asymmetry parameter $q_X = \sqrt{\langle q_\lambda^2 \rangle}/n_X$ today, as a function of the parameters characterising the magnetic field and the vorticity: amplitudes, spectral indices, cutoff frequency and coherence scale.

The amplitude B_λ of a cosmic magnetic field must satisfy stringent constraints, which have been found in previous works using completely independent methods. The most direct constraint on the strength of a magnetic field comes from the observation of Faraday rotation in radio sources: for example, in [32] it is shown that, for a cluster magnetic field, $B_\lambda \lesssim 10^{-6}$ Gauss on scales $\lambda \simeq 1$ kpc. If the charge imbalance, and consequently the magnetic field, was created before recombination, we can apply the limits which come from CMB observations (see for example [33]): in [34] limits were derived, by considering the anisotropies induced in the CMB by gravitational waves, created by the anisotropic stresses of a primordial magnetic field (see Eq. (28) of [34], for $\lambda = 0.1h^{-1}\text{Mpc}$). Finally, in [35], a constraint was found on the amplitude of a magnetic field created before Nucleosynthesis, as a function of the spectral index n , by imposing the Nucleosynthesis bound on the energy density of the gravitational waves induced again by the field (Eq. (33) of [35], also for $\lambda = 0.1h^{-1}\text{Mpc}$).

We next need to find a constraint on ω_λ , which is not present in the literature. Again, it can be found using the isotropy of the CMB. We need to evaluate the vector contribution to the CMB temperature anisotropy, and we use the formalism of [36]. Our vorticity variable $\omega_i = \frac{1}{2}\epsilon_{ijk}\omega^{jk}$ is related to the vector gauge invariant variable $V_C = v^{(1)} - B^{(1)}$, where $v^{(1)}$ is the vector part of the fluid velocity perturbation and $B^{(1)}$ the vector metric perturbation, through relation $\omega_{ij} = a V_C Q_{[ij]}^{(1)}$, where $Q_i^{(1)}$ denotes the vector harmonic [25]. Therefore, $\omega_i = \frac{k}{2}(v^{(1)} - B^{(1)})Q_i^{(1)}$. For the purposes of this analysis, we neglect the finite thickness of the last scattering surface and work in the tight coupling limit, for which $v^{(1)} = v_\gamma = v_B$. In this limit, the dominant effect of vector perturbations is the creation of a dipole in the temperature anisotropy. From the general expression for the CMB power spectrum from vector perturbations given in [36] we finally find

$$C_\ell = \frac{8}{\pi} \frac{\ell(\ell+1)}{a^2(\tau_{\text{rec}})} \int_0^\infty dk \frac{j_\ell^2(k\tau_*)}{(k\tau_*)^2} \omega^2(k), \quad (32)$$

where $\tau_* = \tau_0 - \tau_{\text{rec}}$, and the factor $a^{-2}(\tau_{\text{rec}})$ accounts for the vorticity time evolution. This gives us

$$\frac{\ell^2 C_\ell}{2\pi} \simeq 4\sqrt{\pi} \frac{\Gamma(\frac{3-m}{2})}{\Gamma(\frac{m+3}{2})\Gamma(\frac{4-m}{2})} z_{\text{rec}}^2 \omega_\lambda^2 \frac{\lambda^{m+3}}{\tau_*^{m+1}} \ell^{m+1}. \quad (33)$$

The maximum value for the CMB temperature anisotropy is $\ell^2 C_\ell / (2\pi) \simeq 10^{-10}$, and in order to constrain ω_λ we fix the ℓ dependence by considering the two extreme values $\ell = 4$, if $m < -1$, and $\ell = 200$, if $m > -1$.

We can finally evaluate Eq. (31), and derive an upper bound on the charge density in a region of size λ , by applying the aforementioned constraints on B_λ and the limit on ω_λ just derived. The bounds are represented in fig. 1, as a function of the spectral index

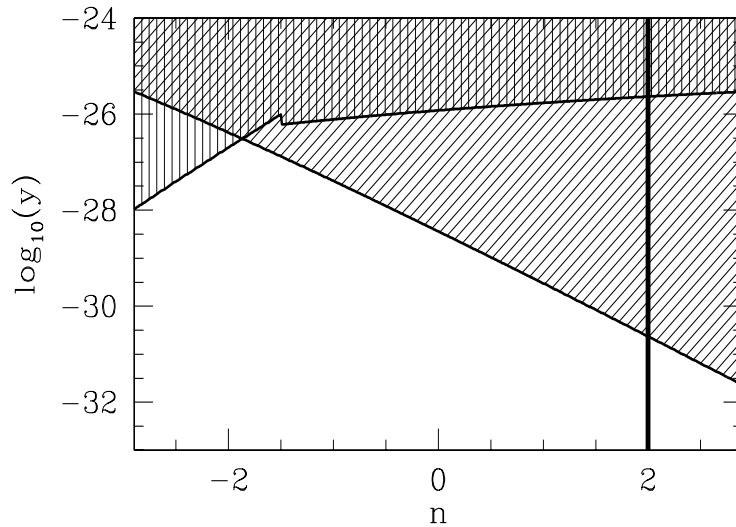


Figure 1. This figure shows the limits on the charge fraction parameter y which apply to a stochastic distribution of charge, as a function of the spectral index of the induced magnetic field, for $\lambda = 0.1h^{-1}\text{Mpc}$. The diagonally-shaded region represents the exclusion region derived from nucleosynthesis constraints [35]; the vertically-shaded region is excluded by CMB anisotropies [34]. Magnetic field constraints from clusters [32] give a less restrictive bound, which is not shown in the figure. If the process generating the charge is causal, then $n \geq 2$.

n of the magnetic field. For every value of n , we have maximised (31) with respect to the vorticity spectral index m , on which we have no theoretical prediction. Note that the quantity shown in the y-axis,

$$y = \frac{q_X}{e} \left(\frac{\Omega_X m_P}{\Omega_B m_X} \right) \quad (34)$$

is the charge fraction parameter ‘normalised’ to the case of ‘baryonic’ charge asymmetric matter (as in Eq. (28)). If the process generating the charge and the magnetic field is causal, *e.g.* it takes place during a phase of standard Friedmann expansion, then the spectral index must satisfy $n \geq 2$ [37]. In this case, the limits on q_X are more stringent than in the case in which the generation occurs during inflation.

5. Discussion

In this paper we have derived a cosmological constraint on the presence of a non-zero electric charge in the universe. A conservation law associated with a long range force, such as charge conservation with the electromagnetic force, is usually believed to be exact as a result of gauge invariance. Conservation of charge appears to hold in all particle decays, and there are strong experimental constraints on the charge of particles which are predicted to be neutral by the standard model. However, even though electric charge conservation is well established on Earth, this may not imply directly the overall

neutrality of the universe: to be able to draw this conclusion, one would have to assume in addition that charge has to be conserved on all scales and during the entire evolution of the universe. This may not be the case and the determination of a cosmological constraint on the charge asymmetry of the universe is of conceptual importance.

There are a host of proposals that lead to the generation of a charge asymmetry in the universe. In the context of brane world scenarios, charge non-conservation arises in the form of charge leakage into extra dimensions; in varying speed of light theories, it can be a consequence of the variation of the fine structure constant; allowing for some modification of the standard model, it can arise if the gauge symmetry is temporarily broken during a phase transition taking place in the early universe, or if one imposes a small, non-zero mass to the photon. In our analysis, we did not consider a complete model in which the charge asymmetry originates: by doing so, our constraints gain in generality. The assumption we made, that charge is conserved in the period in which our constraint is established, points toward a model in which the charge is created during a primordial phase transition leading to a transitory breaking of the electromagnetic gauge symmetry.

We have obtained our constraints on the overall charge imbalance of the universe in two different cases: a uniform distribution of charge, and a distribution characterised by stochastic fluctuations. In order to derive our results, we made the assumptions that the universe is a good conductor, and that the charge is a first order perturbation in a background FRW model; moreover, we generalised the scaling of the charge-induced vorticity as the inverse of the scale factor. These are the only assumptions which affect the uniform distribution limit; in the case of the stochastic distribution, we further assumed that vorticity and magnetic field are two independent, Gaussian stochastic variables. A possible extension of our analysis would be to go beyond the linear calculation, and evaluate the full effect that the presence of a non-zero charge has on the background dynamics of the spacetime.

In the case of a uniform distribution of charge, it is remarkable that our cosmological limit, once translated in terms of constraints for the single charge carriers, gives results which are orders of magnitude stronger than the ones derived by terrestrial experiments or astrophysical observations. If the charge is distributed stochastically, the limits are less stringent: this is a consequence of the fact that the CMB bound on stochastic vorticity is less tight than in the uniform case, and in addition we have maximised it with respect to the vorticity spectral index m . However, we still find interesting constraints for high values of the magnetic field spectral index: these would apply, in the case of a causally created charge asymmetry, for example during a phase transition.

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